**Study 1: Place value understanding enhancement using the Japanese abacus**

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This preliminary study reports on the impact of an 11-week intervention programme using the Japanese abacus as a manipulative tool on twelve underperforming Years 3 and 4 children's place value understanding, computation strategies, and arithmetic scores. The effectiveness of the programme was measured against a control group of thirteen better performing students using Chan et al.'s (2014) place value assessment test, and self-designed arithmetic tests. The experimental group's scores improved on average following intervention by 89% in arithmetic and 49% in place values. These were significantly better than the improvement in performance of the control group, whose results rose on average by 25% and 4% respectively - although this may partially reflect a ceiling effect on possible test scores. A noticeable shift in counting strategies from unitary and finger counting to the use of more sophisticated counting methods including mental computations by the intervention group is evident after the intervention. These findings further support earlier research which has shown that physical manipulation enhances children's arithmetic abilities.

Keywords: word; place value, arithmetic, abacus, mental computations, KS1

## Introduction

Place value refers to the value attached to a digit depending on where it sits in a string of numbers. It is considered to be one of the building blocks in children's mathematical journeys. Hence, it is usually introduced fairly early in primary schools across different continents. A research by Thompson & Bramald, 2002, shows that only half of the UK Year 2 children in their sample knows that the 1 in 16 refers to a ten despite of the UK national curriculum recommendation for it to be introduced as early as in Year 1. The lack of a good place value understanding has been linked to mathematics learning difficulties by various independent researchers (Chan, Au, & Tang, 2014; Moeller, Pixner, Zuber, Kaufmann, & Nuerk, 2011; Chan & Ho, 2010; Hanich, Jordan, Kaplan, & Dick, 2001; Jordan & Hanich, 2000; Miura & Okamoto, 1989). In particular, it is noted that children who fall within this category tend to make mistakes on sums involving addition and subtraction with regrouping (Fuson, 1990).

This study explores the role of the abacus as a manipulative tool in bridging children's understanding of abstract concepts such as the place value. While the use of the abacus has been widely explored to enhance children's mental computations (Barner, et al., 2016) , encourage positive attitude to school maths (Shwalb, Sugie, & Yang, 2005), increase concentration spans , and develop visio-spatial ability (Wang, Geng, Yao, & Wen, 2015), the author knows of no other study which has looked into the use of the soroban in enhancing place value understanding, with a specific focus on underperforming children. Within this study, children are taught how to use the abacus to perform simple arithmetic computations in the first six weeks. They are then, encouraged to attempt completing simple double digits sums mentally through the visualisation of the abacus beads. Towards the latter part to the study, children explored larger number place values and learned different strategies in completing single, double, and multi-digit addition with regrouping without relying on the use of the abacus.

Subtraction with regrouping was covered towards the last three weeks of the programme. All children were discouraged from finger and unnecessary unitary counting following from (Cotter, 2000) finding that unitary counting can interfere with children's place value understanding and result in rote learning. She notes that Japanese children are discouraged from unitary counting from the age of six. Geary, Hoard, Byrd-Craven, & DeSoto (2004) also find that the over-reliance of the unitary counting method has often been found in children who have mathematics learning difficulties.

According to Fuson, et al. (1997), children's double digit place value conception consists of five stages. They usually start from a 'Unitary' stage where they rely on counting in ones to arrive at a final figure. They will, then, move on to the 'Decades and ones' stage when they understand that numbers such as 35 can be broken down into 30 and 5. Following this, they will not only partition numbers into tens and units, but are also able to count in sequences of tens before counting in ones (10, 20, 30, 31, 32, 33, 34, 35). This is also known as 'Sequence tens and ones' stage. As children improve further, they also recognise that 35 is comprised of 3 tens and 5 ones, also known as the 'Separate tens and ones' stage. Finally, children will enter an 'Integrated sequence-separate-tens' stage where they will have the ability to use either of the sequencing of tens and separate tens interchangeably. Developing a sound understanding of place value can help children partition numbers easily and enable them to apply the knowledge to complete any mental addition and subtraction more readily. 66 + 27, for example, can be easily completed by partitioning the 66 into 60 and 6, and the 27 into 20 and 7. Adding units to the units (6 + 7) makes 13. Adding tens to the tens (60+ 20) makes 80. The answer is then 80 + 13 is equal to 93.

The superiority of place value understanding among East Asian children in comparison to Western counterparts is well documented. Some attributed the superiority in place value understanding to native languages used in these countries. They more naturally map the place value structure of the Arabic numerals, allowing for a better understanding of how numbers slide into different places (Miura, Okamoto, Kim, Steere, & Fayol, 1993).

Others have explored underlying cultural factors, such as the widespread use of the abacus, an ancient counting tool, within local school curriculums and as part of after school activities. Shwalb, Sugie, & Yang ( 2005), for example, reported that while the use of the soroban within formal schooling has been limited to Grade 3 as an exposure to Japanese traditional culture, it is widely studied by many children as part of after school clubs.

**Study 1**

This is an exploratory study investigating the effectiveness of using the Japanese abacus in conveying place value concepts to underperforming children at the primary school level. It also examines whether greater place value understanding leads to changes in counting strategies used when attempting sets of multi digit addition and subtraction questions at pre- and post- intervention sessions.

The study was conducted over a period of 11 weeks to 12 underperforming children in Years 3 and 4 of a UK primary school. The school is a smaller than an average size primary school with the majority of students to be White British. It has an above average proportion of students who are supported by pupil premium funding as well as have special needs statements. Two one-hour sessions were run on a weekly basis in two small groups. As part of the study, some children from each of the Years 3 and 4 were selected to participate as control groups. The control groups did not receive an alternative intervention method as the intervention sessions often coincided with class maths hours. All children in both groups were selected by their class teachers.

**Method**

In the first six weeks, children learned how to operate the abacus and used it during this period to complete sums given. They worked on partitioning double digit numbers into tens and units. Numbers in their teens were particularly focused on before exploring other larger numbers. This exercise was necessary to eliminate the pre-conception that 13 consists of one and three instead of a ten and three units. Children also looked at different number sequences using different starting points to encourage flexibility in counting. They also explored the inverse relationship between addition and subtraction through exercises such as finding out the difference in value between two numbers. Lastly, we covered how they might complete two double digits additions mentally following from their understanding of double-digit numbers partitioning. During the sessions, children were always encouraged to explain how they solved a particular question and at times, to demonstrate to the others.

In the final five weeks, they initially explored place values into hundreds, thousands, and at times into millions using the abacus, but would be subsequently asked to name the value of each of the digits in a string of number without relying on the abacus. They were shown different strategies in completing single digits with regrouping. Finally, they also spent approximately 3 sessions exploring 2-digit subtractions with regrouping. Children were shown how the borrowing and carrying worked on the abacus but they were not relying on the abacus while completing tasks involving different sums.

Three different strategies for solving single digit additions with regrouping were explored. For example, 7 + 8.

- The first method involved using the understanding of the doubling of single digit numbers. We worked out the double of sevens to make 14 and added one more to make 15.
- The second method which is the basis of the abacus operation will require number bond 10 understanding. We partitioned the 7 into 5 and 2 and left the 8 as it was, so that 5 + (2 + 8) = 5 + 10 = 15 or partitioned the 8 instead into 5 + 3.
- The last method is also shown using the abacus, involving partitioning any numbers larger than 5 into 5 plus some units. We partitioned 7 + 8 as (5 + 2) + (5 + 3). We, then, re-arranged them into (5 + 5) + (2 + 3), which made 10 + 5 = 15.

Children were encouraged to implement the methods they were most comfortable with as finger counting was banned at all time. 'No finger counting' rule was successful in, firstly, stopping children relying on unitary counting, and secondly, encouraging them to find different solutions to the questions posed to them. All children adopted the rule promptly and regulated each other during sessions to ensure nobody finger counted. When children were familiar with single digit additions, they were encouraged to attempt higher number additions to hundreds and thousands through any methods they were most comfortable with following from demonstrations.

For subtraction with regrouping, we used the following strategy. When completing 13 – 6, for example, they were encouraged to partition 13 into a ten and three units as clearly demonstrated using the abacus. As the three was not sufficient to take away six, leave the three to one side, untouched. Instead, they could use or borrow the ten to take away the six (10 – 6 = 4). They, then, added on the four to the existing three to make seven. This method made mental computations possible without relying on finger counting. In order to perform this method, however, children would need to be familiar with number bonds 10 and knew how to partition 2-digit numbers into their tens and units. See Appendix 1 for lesson plans.

The pre- and post-intervention assessments consisted of arithmetic and place value assessments. The arithmetic assessments were designed by the author, consisting of 3 parts. The first part looks at the children's ability to write down five of each 2-, 3-, and 4-digits numbers dictated to them. The second and third parts consist of 15 horizontal and 10 vertical sums up to thousands respectively. This is to see if children's answers are affected by the layout of the questions. See Appendix 2 for arithmetic test paper.

The place value assessment was adapted from (Chan, Au, & Tang, 2014) which extends the standard two-digit place value to three-digit place value understanding tests on children in Hong Kong. The test consists of graphical presentations of individual squares, towers of 10 squares, and big blocks of 100 squares. Children are asked to count the number of squares presented in different orders such as towers of tens and individual squares; blocks of hundreds, towers of tens, and individual squares; blocks of hundreds and individual squares; blocks of hundreds, individual squares, and towers of tens. See Appendix 3 for place value assessment paper.

***What is the Japanese abacus?***

The Japanese abacus (also known as 'Soroban'), is a base-ten counting tool which allows children to perform arithmetic computations such as addition, subtraction, multiplication, and division. It has one top and four bottom beads on each rod and was adapted from the Chinese abacus which has two top and five bottom beads. The elimination of one of the top and bottom beads enables learners of the soroban to visualise mental computations at great speed and accuracy. The soroban maps the structure of the place value, hence, can be used to complement the column teaching methods taught at primary schools.

Counting on the soroban always starts from any rod with a dot which is the 'Units/Ones' rod. Subsequent values of the beads to the left of the 'Units' rod is 10 times larger than the value of its immediate right hand column rod. Figure 1 shows that if we were to decide that the middle rod is the starting 'Units' rod, to the left of the 'Units' rod is the 'Tens' rod. To the left of the 'Tens' rod is the 'Hundreds' rod and so on. This maps the notations used often within UK classroom of HTU (denoting 'Hundreds', 'Tens', and 'Units') for a 3-digit number.

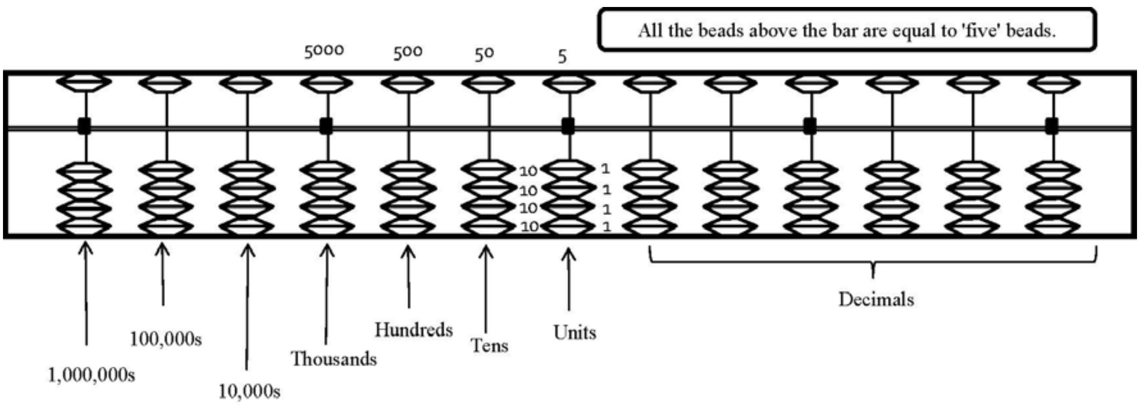
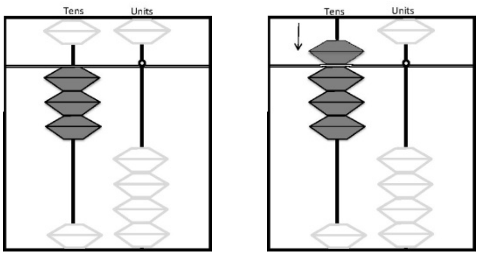



**Figure 1. The anatomy of a soroban**

Additions or subtractions can be completed by pushing the relevant beads towards or away from the middle horizontal bar which divides the top and bottom beads. Every top bead's value is five times that of each of the bottom beads on the same rod. For example, the top bead in the 'Tens' rod is worth 5 x 10 = 50. Each of the bottom beads is worth10. Pushing 3 bottom beads towards the horizontal bar on the 'Tens' rod will imply that we are adding 30. If we were to push the top bead of the same rod down towards the horizontal bar, we are adding another 50. Hence, altogether we are counting 80 (see Figure 2). Note that the soroban can also be used to teach decimal places.  As soon as children understand how to operate the soroban, the transition to adding or taking away on the decimal places is straight forward.



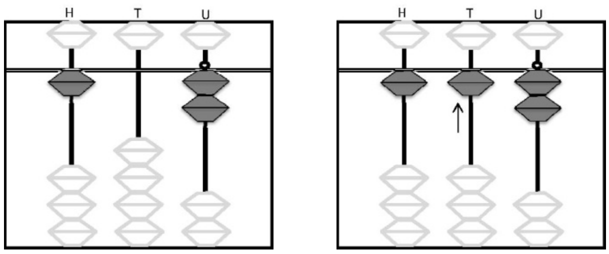




**Figure 2. Counting 30 and add on 50 to make 80.**

**Figure 3. Counting 102 and add on 10 to make 112.**

The soroban allows children to add on beads according to their place value visually.  It can be useful in demonstrating that adding tens can be done on the tens rod without having to go through any sequences of ones to arrive at the answer.  For example, the soroban can clearly demonstrate that adding a ten to 102 will make 112 (see Figure 3). The interpretation of what 112 consists of is straight forward as children can clearly see that it is made of one bead of the hundreds, one bead of the tens, and 2 beads of the units.  In addition to this, the soroban can be a useful tool in demonstrating how the numbers should be written compared to how it is sounded.  For example, one hundred and two is often misunderstood as 1002.  The soroban can quickly clarify this misunderstanding as it allows children to partition 102 into 100 and 2.

## Findings

The results are presented based on year groups (years 3 and 4) with a number assigned to each individual student.

### *Year 3 pre- and post-arithmetic results*

Results from Figure 4 (Pre Arith) shows that the control group children (CG) started at a much higher base than the intervention group (IG).   The majority of the scores the IG achieved at the pre-intervention stage are from the dictation section of the test.    All Year 3 IG children were able to write down the 2-digit numbers read out to them, but the majority were unsure of numbers in their hundreds and thousands.  On the arithmetic computations for both the vertical and horizontal formats, most children only attempted on average 5 out of 25 numerical questions. None of the IG children could answer simple double digit additions such as 12 + 12. Most questions were left with no attempt at solving them.

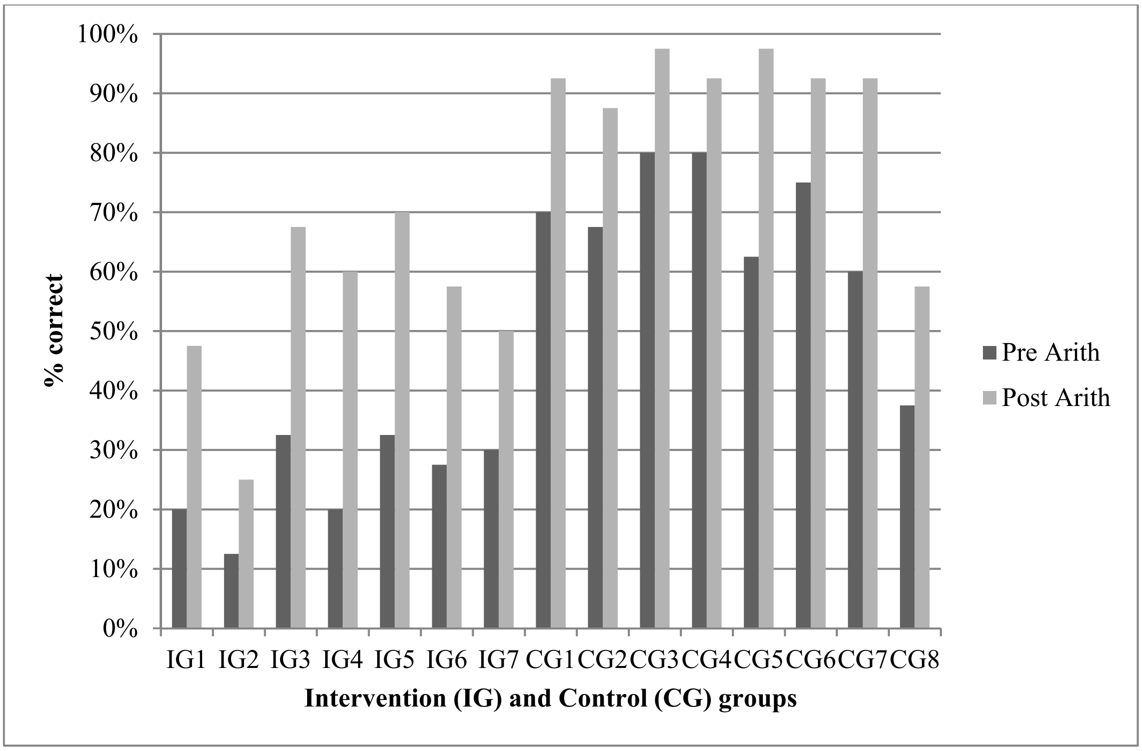



**Figure 4. Year 3 pre- and post-arithmetic results**

Following from the intervention (see Post Arith), the majority of IG children demonstrated a better understanding of numbers dictated to them. Some were able to write down dictated numbers in the thousands. The author noticed surges in confidence in IG children evidenced by the fact that all but one child attempted almost every question in the arithmetic test. The majority of children did very well in their additions. Some of them were able to answer questions such as 468 + 243 and 620 + 620 correctly. Subtraction with regrouping, on the other hand, was a challenge to many of the IG children in this year group which could be partly due to the fact that we only spent a small fraction of the intervention time on this topic. No children used any finger counting in attempting the test, although they were allowed to use any methods in their computations.  All children but IG2 attempted the questions mentally with very minimal reliance on the column addition method.  While IG2 did not perform as well as the other children, there was a definite indication of growing place value understanding from visible lines drawn joining units to units and tens to tens when adding sums.  This is a substantial step from not recognising the 'take away' sign prior to the intervention.

All CG children demonstrated a good understanding of dictated numbers up to 4 digits at pre-intervention.  Column addition method with HTU notations along the top was noticeably used by most children in this group.  Many of these children were able to work double digit sums without regrouping mentally.  There was only one child (CG8) within this group who still relied on unitary counting.  This was visible from the sequence of numbers written all over the test sheet to arrive at the answers (see Figure 5).  There is no noticeable effect in either group's arithmetic scores due to the different layouts (horizontal and vertical) of questions presented at the pre- and post-intervention tests.

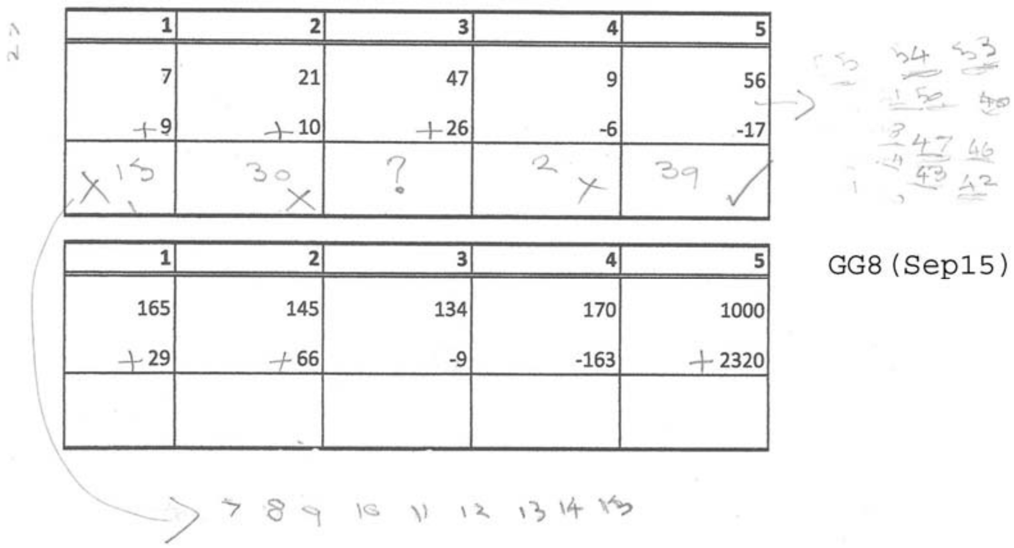



**Figure 5. Unitary counting applied by CG8  at the pre-intervention test**

All CG children found subtraction with regrouping questions challenging at the pre-intervention period.  The majority, though, overcame the challenge with the exception of CG8 who continued to apply the unitary counting method at the post-intervention test. ‘Count all’ strategy was used with the help of tally marks to add up two single digits with regrouping. On subtraction with regrouping (15 – 7), CG8 would place 15 tally marks and crossed 7 tally marks out to arrive at the answer. Without examining the counting strategy used, CG8 appeared to have done as well as most of the children in the IG. On closer inspection, though, it can be argued that CG8 had not moved beyond the unitary counting strategy. This could potentially hold CG8 back in developing further in his arithmetic learning journey.

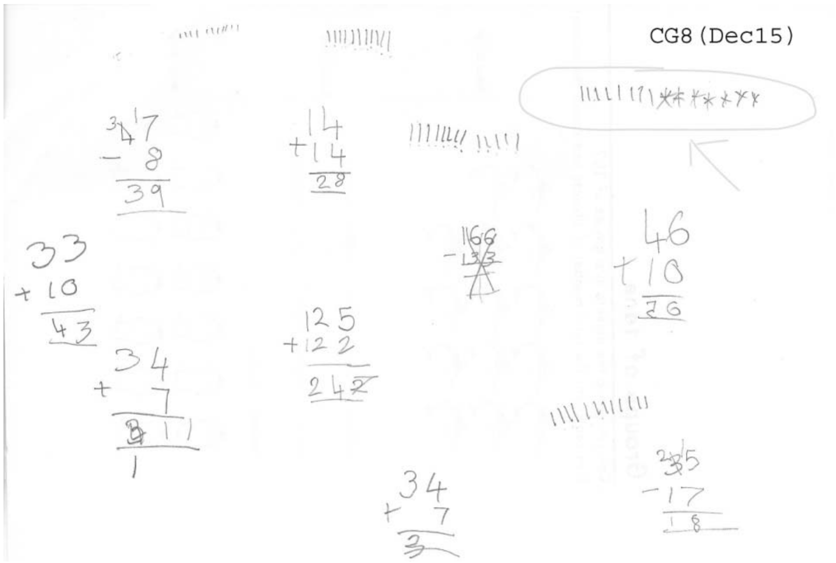



**Figure 6. Tally marks used by CG8 at the post-intervention test**

### *Year 3 pre- and post-intervention place value test results*

On the pre-intervention place value test (Pre-PV), the CG clearly demonstrated a better understanding of place value. Many children were able to count in ones, tens, and hundreds with ease. This was not the case for the IG children. Two of the children couldn’t answer any of the questions. Some children were counting all the squares individually even though they were in towers of tens and blocks of hundreds. There was a clear delay in adopting more advanced counting methods which given their arithmetic performance possibly supports existing literature linking immature place value understanding and mathematical difficulties.

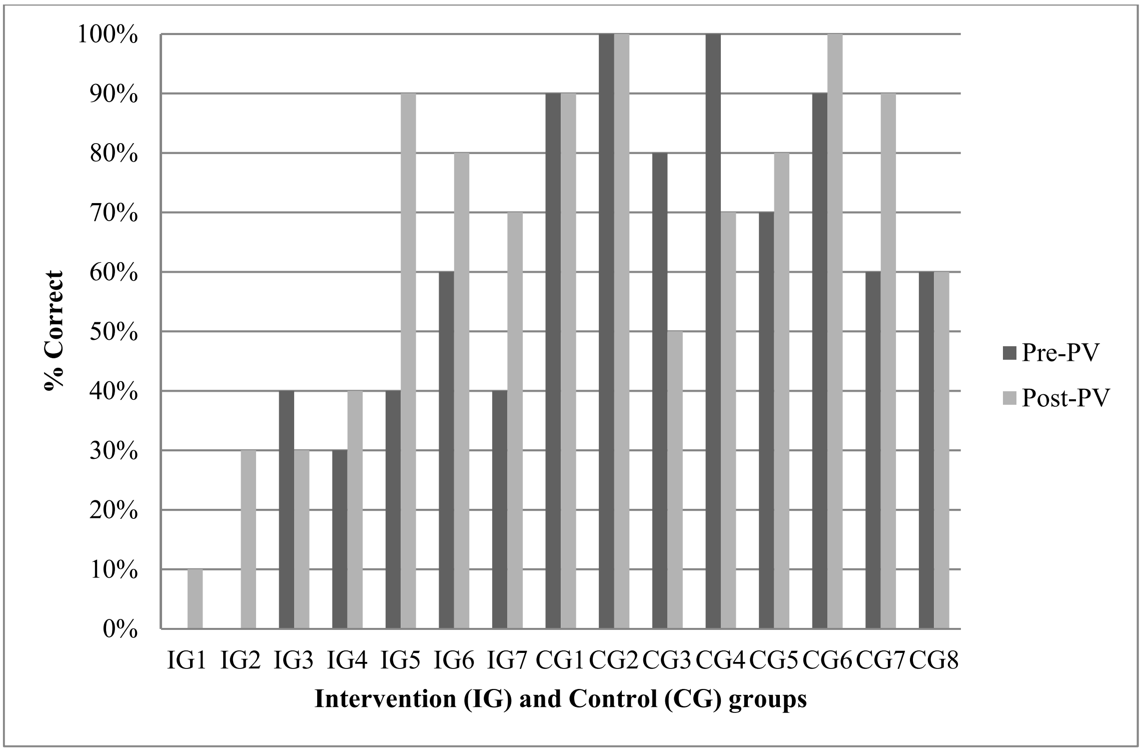



**Figure 7. Year 3 pre- and post-intervention place value test results**

At post-intervention (Post-PV), there were clear indications of shifts in counting strategies from unitary to later place value developmental stages as specified by Fuson (1997) in that some IG children were seen to be applying sequences of hundreds and tens in arriving at the answers.

### *Year 4 pre- and post-arithmetic results*

Most of the Year 4 IG children demonstrated a better knowledge of dictated numbers with two of them showing a good understanding of numbers into the thousands at the pre-intervention period (Pre Arith). Following from the intervention (Post Arith), all children showed a better grasp of dictated numbers into the thousands. All CG children, on the other hand, scored almost fully on pre- and post-intervention dictation tests.

On the arithmetic tasks, the disparity between the two groups' numerical abilities was not as stark as the groups in Year 3. There was, however, still a clear difference in the two groups' abilities. CG children showed a strong reliance on column addition methods at pre- and post-intervention periods. None of the CG children attempted the questions mentally.

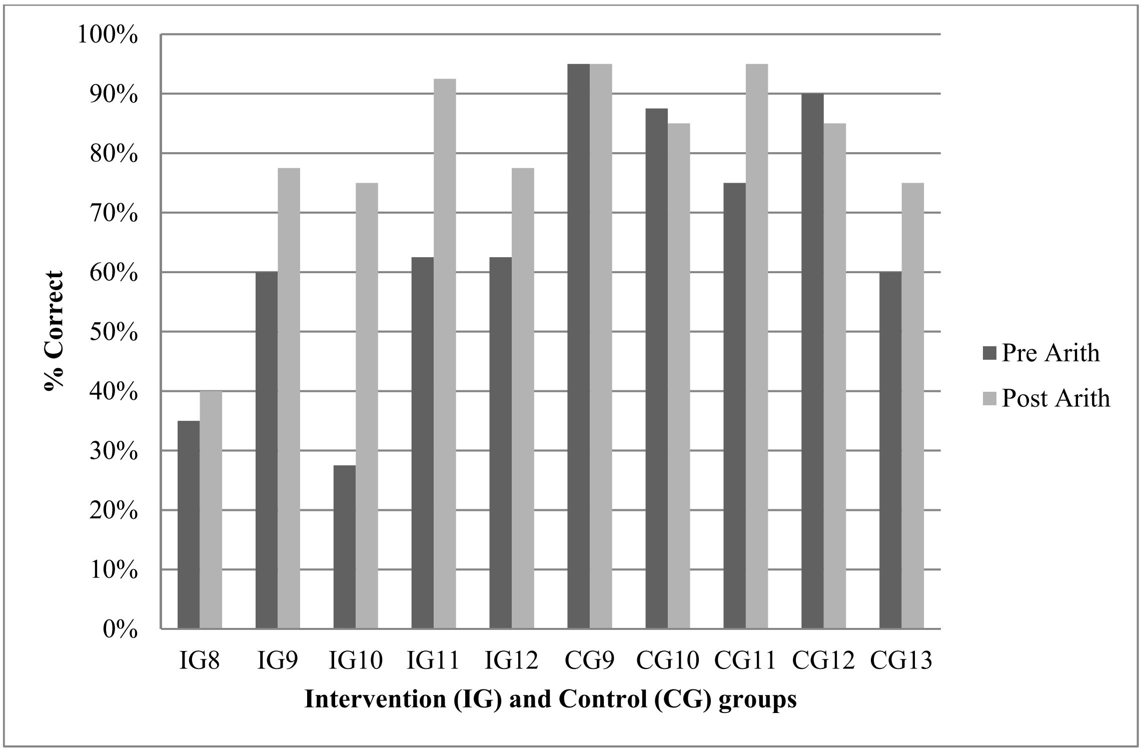



**Figure 8. Year 4 pre- and post-arithmetic results**

Within the IG, however, the intervention seemed to have had a significant impact on IG10 and IG11 in particular. IG10 who managed to only answer one arithmetic question (10 + 7) at pre-intervention, attempted all questions mentally at post-intervention. IG11 previously counted on 30 ones on her fingers to complete 26 + 30 at the pre-intervention assessment, was able to move beyond unitary counting and scored as well as the top performing children in the CG. This, further, supports the importance of place value understanding. IG8 was the lowest performing of the group and is a special needs child. She was included as part of a long term intervention programme.

***Year 4 pre- and post-intervention place value test results***

On place value tests, all but one of the IG children displayed an improvement in their place value understanding. This is in contrast to the CG children where only one child performed better at the post intervention period. On average, however, the level of the

IG children understanding did not surpass that of the CG children. The different layouts (horizontal and vertical) of questions presented at the pre- and post-intervention tests do not affect any of the Year 4 children's arithmetic scores.

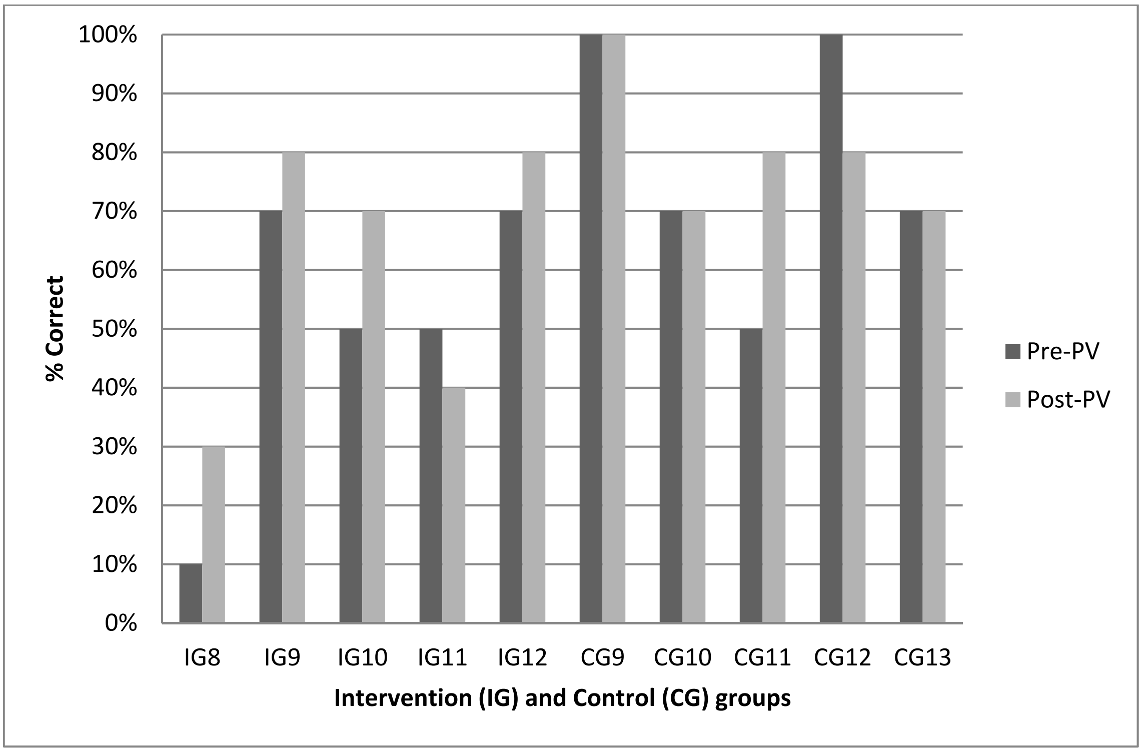



**Figure 9. Year 4 pre- and post-place value test results**

## Data analysis

Non-parametric tests are applied to allow for a more appropriate analysis without relying on normality assumption due to small sample size. Wilcoxon paired t-test is used to see whether the intervention and teaching at school have been effective in improving children's place value understanding and arithmetic scores. Following from this finding, Mann-Whitney test is used to examine whether any of the two groups significantly outperforms the other. Lastly, Spearman's rank correlation is applied to measure the strength of association between the place value and arithmetic scores.

Wilcoxon test uses W statistics to accept or reject the null hypothesis that there is no difference in the median and mean between the pre- and post-intervention scores. A lower W statistics than the critical values implies the rejection of the null hypothesis

Please see users.sussex.ac.uk/~grahamh/RM1web/WilcoxonHandoout2011.pdf for steps in conducting the test.

| | Arithmetic W Statistics | N | Critical values (significance levels) | Place Value W Statistics | N | Critical values (significance levels) |
|---|---|---|---|---|---|---|
| IG | 0 | 12 | 7 (1%) | 7 | 12 | 10 (2%) |
| CG | 3 | 12 | 7 (1%) | 14 | 7 | 2(5%) |

**Figure 10. Wilcoxon paired t-test on whether the mean/median between the pre- and post-intervention scores equals to zero**

Figure 10 shows that children in the IG have benefitted from the intervention in both their arithmetic skills and their place value understanding at the 1% and 2% significant levels. The CG has equally shown improvement due to the classroom teaching at 1% significant level. Place value understanding for this group is, however, found to be insignificant which could be due to ceiling effect in the test set.

A further test is conducted to see if any improvement seen in the IG does outperform the CG despite both groups having benefitted from the intervention and classroom teaching. Mann-Whitney test is used to analyse the hypothesis. Please refer to users.sussex.ac.uk/~grahamh/RM1web/MannWhitneyHandout%202011.pdf for steps in conducting the test.

We should expect significant difference between the two groups' rank totals if there is indeed a significant difference in the results between the two samples of the study. We should see one group of students' scores' ranking to be high and the other group's ranking to be low on the spectrum. If there is no significant difference in the two groups' performance, there will be less pronounced difference in the two groups' rank totals. As the test relies on rank totals to compute the U statistics, it is a more robust test compared to the t-test as it is less likely to indicate significance due to chances because of outliers. Smaller U statistics than the critical values indicate a rejection of the null hypothesis that the IG doesn't outperform the CG in each of the arithmetic and place value scores.

| Score differences in | Computed U scores | Critical values (significance levels) | Z-scores (p-value) |
|---|---|---|---|
| Arithmetic | 46 | 47 (10%) | -1.71337(0.0867) |
| Place value | 40 | 41 (5%) | -2.03973(0.4145) |

**Figure 11. Mann-Whitney test on whether the intervention groups outperform the control groups**

Figure 11 shows that the improvement in arithmetic and place value scores achieved by the IG children (both years 3 and 4) are found to be significantly different from the CG (both years 3 and 4) at the 10% and 5% confidence levels respectively. The results are encouraging given the fact that the children within the IG started at much lower bases than the CG children.

| | Pre-intervention | Post-intervention |
|---|---|---|
| IG | 0.65 | 0.57 |
| CG | 0.59 | 0.22 |

**Figure 12. Spearman rank correlation between place value and arithmetic scores**

On the correlation test, the IG displayed a fairly strong positive co-movement between the place value and their arithmetic scores at 0.65 and 0.57 respectively at the pre- and post-intervention tests. The correlations at pre- and post- intervention for the CG children, on the other hand, are found to be 0.59 and 0.22 respectively. This could be a reflection of a ceiling effect where children who have scored well at the pre-tests were not able to score further at the post-tests. This was more noticeable for the Year 3 than Year 4 control group children.

**Conclusion**

The findings in this study provide some evidence of the effectiveness of place value understanding enhancement using the Japanese abacus at the primary school level. Understanding place value has been a problem particularly among children whose languages do not map the structure of the Arabic numbers, including English. Children from East Asian countries such as China, Japan, Korea, and Taiwan are found to acquire a good understanding of place value concepts before they even start formal schooling. The use of the Japanese abacus within this study has helped in conveying the concept which is evidenced by the application of more sophisticated counting strategies and the improvement in arithmetic scores at the post-intervention test. The effectiveness of the study is further supported by the data analysis which confirms the outperformance of the intervention group against the control group. Similar finding has also been suggested by Collet (2003) that the use of base-ten manipulative encourages mathematical problem solving.

Given the wider research into the benefits of using the soroban conducted, by among others, (Barner, et al., 2016), and the results presented in this study, there is a case for exploring the use of the abacus further on a larger scale to allow a more robust data analysis into its effectiveness. There might be important and timely benefits from using the Japanese abacus in UK primary school classrooms, especially given the emphasis on place value understanding in the latest UK Primary Mathematics Curriculum. The use of the soroban in young English speaking children will not only allow children to explore the different place value developmental stages, but also compliment the maths curriculum.

***Limitations***

The soroban has helped the majority of the children in the study, with the exception of the very weak ones. It is, though, possible that a longer intervention period is required to help these children. Three of the IG children have carried on with their intervention post the conclusion of the study. They have shown progress in their mental computations following from a more consistent use of the abacus over a longer time period. Given this, there is a case for implementing the full use of the abacus as a manipulative tool for all computational tasks instead of using it as a mere place value enhancement tool. It will also be beneficial to focus more fully on multi-digit additions and only include subtractions when a longer period of study is feasible. There is also an issue of small sample size. The study will benefit from bigger sample size where underperforming children are randomly assigned to the intervention and control groups in which the IG will adopt the use of the abacus only and the CG will adopt other tools using the same materials.

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**Appendix 1. 11-week lesson plans**

| | |
|---|---|
| WK1 | Introduction to the Japanese abacus and tens and units. |
| WK2 | Place value up to 20, number magnitude and sequences, adding single digits sums mentally. |
| WK3 | Place value up to 30, mental computations of tens and units, adding/taking away two digits sums using the abacus, and number bonds 5. |
| WK4 | Place value up to 100, number magnitude and sequences, computations of double and single digits which sum up to less than 20, number bonds 5. |
| WK5 | Place value beyond 100, working on double digit sums. |
| WK6 | Explore double digit sums of less than 50 (non carry), number bonds 10. Single digits additions with regrouping, explore a + b = c, therefore, a = c - b and b = c - a. |
| WK7 | Double digit additions with regrouping, mental computations of double digit sums up to 50. |
| WK8 | Place value in hundreds and thousands, double digits sums with regrouping, encourage mental computations involving sums less than 100. |
| WK9 | Multi-digit sums with regrouping beyond 100 and mental computations of two single digits sums with regrouping. Double digit subtractions without regrouping. |
| WK10 | Multi-digit sums with and without regrouping. Place value in hundreds and thousands. Double digit subtractions with grouping. |
| WK11 | Place value in the thousands, subtractions with regrouping. |

***Appendix 2. Sample of Arithmetic test paper for the post-intervention period***

Note that the numbers within the dictations part are the numbers being dictated to the children.

| | |
|---|---|
| Date : | |
| Name: | |

| Dictations | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 2 digits | 80 → | 17 → | 56 → | 79 → | 99 |
| 3 digits | 108 → | 115 → | 160 → | 175 → | 186 |
| 4 digits | 1005 → | 1070 → | 3650 → | 3366 → | 4515 |

Horizontal format questions

| | | |
|---|---|---|
| 10 + 5 = | 73 - 20 = | 100 + 70 + 7 = |
| 14 + 14 = | 47 - 8 = | 125 + 122 = |
| 46 + 10 = | 35 - 17 = | 468 + 243 = |
| 34 + 7 = | 108 - 60 = | 620 + 620 = |
| 36 + 26 = | 166 - 133 = | 1007 + 50 = |

Vertical format questions

| 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|
| 4<br>+ 8 | 33<br>+ 10 | 58<br>+ 24 | 8<br>- 5 | 63<br>- 15 |
| | | | | |

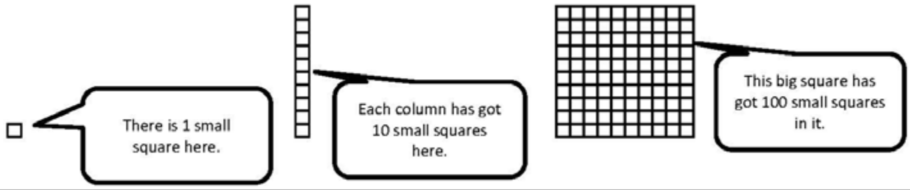
| 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|
| 112<br>+ 19 | 155<br>+ 55 | 142<br>- 9 | 180<br>- 151 | 1000<br>+ 3455 |
| | | | | |

***Appendix 3. Place value test adapted from (Chan, Au, & Tang, 2014)***

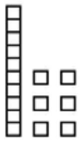
Note that children were initially asked to work out a sample question (located at the top of the test paper) to ensure that they understood what was asked of them.

Name: Date:

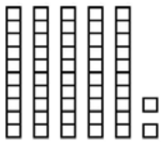
Here are some information to help you with the counting of small squares.



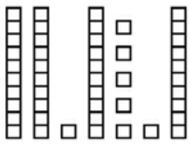

1. How many small squares are there? ____________________



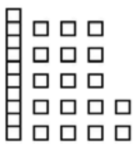
2. How many small squares are there? ____________________



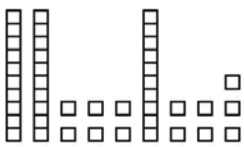
3. How many small squares are there? ____________________



4. How many small squares are there? ____________________

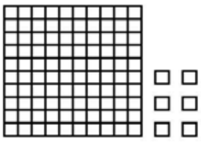


5. How many small squares are there? ____________________

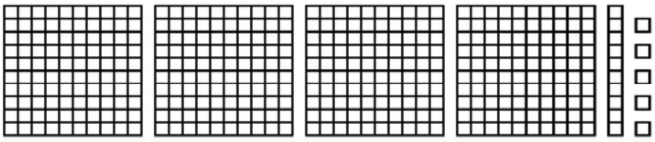


Name: Date:

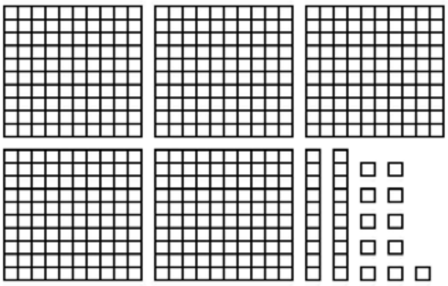
6. How many small squares are there? ______________________



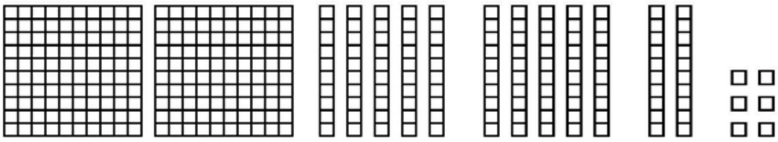
7. How many small squares are there? ______________________



8. How many small squares are there? ______________________



9. How many small squares are there? ______________________



10 How many small squares are there? ______________________