An alternative tool in teaching place value concepts

Nisih Freeman discusses her experience of using the Japanese abacus in teaching UK primary school children place value concepts and the subsequent effect on their arithmetic abilities.

Introduction

Place value is considered to be one of the building blocks in children's mathematical journeys. It is usually introduced fairly early in primary schools across different continents. In the UK, the concept is introduced as early as Year 1. Despite this, research shows that only half of Year 2 understand that the 1 in 16 refers to a ten (Thompson & Bramald, 2002). Various independent researchers have linked, among other things, immature place value understanding and mathematical difficulties. They find that children who struggle with place value concepts have the tendency to make carrying over and borrowing mistakes in additions and subtractions respectively. In early years, this can lead to poor performance in simple arithmetic computations which, if not remedied, can affect both confidence and later years' mathematical achievements.

An alternative counting tool

I have been helping children from years 1 to 5 who are considered to be underperforming by their class teacher in maths over the past few years at primary schools in the UK. The interventions are conducted with the help of the Japanese abacus (also known as 'soroban) as a counting tool.

The Japanese abacus which has one top and 4 bottom beads on each rod, is a close cousin of the Chinese abacus. It has been around for over 400 years. It is a base-ten counting tool which enables children to complete additions, subtractions, multiplications, and divisions. It is also found to encourage mathematical problem solving. A million Japanese children learn the soroban every year despite of the nation being considered a leader in technology advancement (Bellos, 2012). It is a well known tool across many East Asian countries which have dominated maths rankings over the past 20 years (TIMSS, 2016).

The soroban maps how written Arabic numerals can be visualised; see Figure 1. It allows children to dissect multi-digit numbers into their respective columnar values, hence, enables the various stages of double digit place value conception (Fuson, et al., 1997). Children usually start from a 'Unitary' stage where they rely on counting in ones to arrive at a final figure. They will, then, move on to understand that numbers such as 35 can be broken down into 30 and 5. Following this, they will not only partition numbers into tens and units, but are also able to count in sequences of tens before counting in ones (10, 20, 30, 31, 32, 33, 34, 35). As children improve further, they also recognise that 35 is comprised of 3 tens and 5 ones. Children are considered to gain a good place value understanding when they are able to use any of the above partition strategy interchangeably.

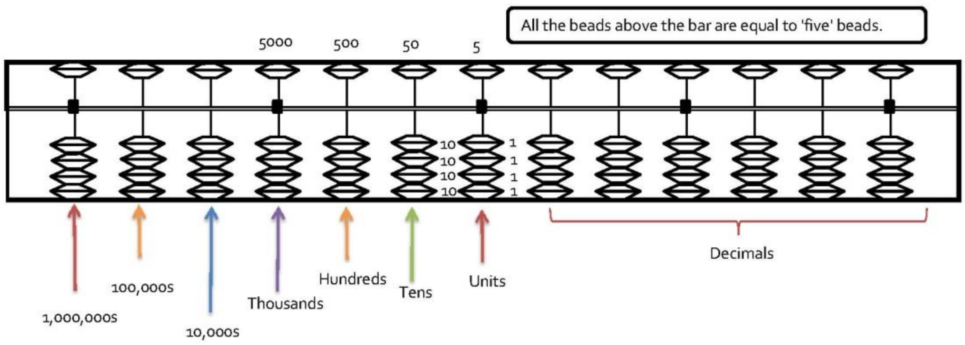
The ability to move on beyond the unitary stage is an essential step. From the age of six, children in Japan are discouraged from counting in ones. Unitary counting interferes with place value concepts and can result in rote learning. Moving beyond counting in ones does not only help children to explore arithmetic computations from different perspectives, but also encourages flexibility in numerical concepts. It can be particularly useful in mental computations. For example, children can complete 34 + 47 by using one of the partitioning strategies. They can think that 30 + 40 makes 70 and 4 + 7 makes 11. The answer is then 70 + 11 is equal to 81. The over-reliance of the unitary counting method has often been found in children who have mathematics learning difficulties. In order to encourage children to move beyond the unitary stage, alternative methods have to be provided to enable them to progress further. Otherwise, those who do not understand how to partition 2-digit numbers will continue to count in ones as can be seen from a Year 4 child who participated in one of the intervention programmes. The child counted in ones 30 times to complete 25 + 30. It was obviously felt to be the best method to obtain the answer. There was no visible attempt to use any other methods taught at schools in solving the question as seen among peer group.

Operation of the soroban

The counting on the soroban always starts from any of the rods with a dot. This is the 'Units/Ones' rod. Once the starting rod is determined, the remaining rods will carry their respective values. For example, to the left of the 'Units' rod is the 'Tens' rod and to the left of the 'Tens' rod is the 'Hundreds' rod.

Push the relevant beads towards the middle bar if adding and push them away from the bar if taking away. Note that every bottom bead below the middle bar represents one count of the value where it sits in the columns (rods). For example, pushing 4 bottom beads upwards on the "Hundreds" rod means we are counting 400 as each of the beads has got a value of 100. Every top bead above the middle bar is worth five times more than those of bottom beads. For example, pushing the top bead downwards on the 'units' rod means we are counting a five. When children are familiar with the values of the beads to the left of the 'Units' rod, the introduction of decimal places will be straight forward using the soroban.

Figure 1: The anatomy of a soroban



I have listed some examples below to enable a better understanding of how the soroban works. Figure 2, for example, shows that we are counting four as there are four 'Units' pushed towards the middle bar. If we were to push the top bead down on the same rod, we are adding a five onto the existing four units to make nine altogether as shown in the next image. Figure 3 is counting 13 as we push one bead of 'Tens' and three beads of 'Units' towards the middle bar. As children learn each place's values, they will be able to partition numbers or add on numbers accordingly. For example, children can easily see that 13 is made of a 10 and 3. If they need to add on another 10, they can quickly add on a bead of 'Tens' on the 'Tens' rod without having to count 10 'Units' to arrive at 23. Figure 4 is counting 67 which is made of 60 (50 + 10) and 7 (5 +2). Figure 5 is counting 303 where children can partition the numbers accordingly to make 300 + 0 + 3. It can also be used to add on any hundreds or tens or units in different orders.

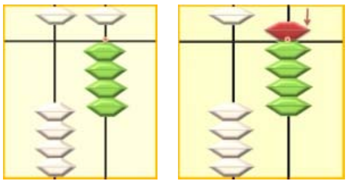


Figure 2: Counting four and add on 5 to make 9.

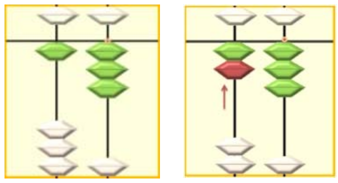


Figure 3: Counting 13 and add on 10 to make 23.

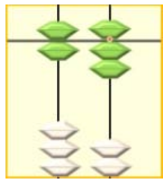


Figure 4: Counting 67

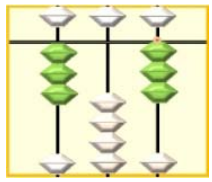


Figure 5: Counting 303

Intervention structure

The intervention structure described below is a broad structure of what children from Years 1 to 5 go through when they participate in intervention programmes.

Place value: In the initial stage, children explore the values of different beads shown to them, in particular, the partitioning of two-digit numbers into their tens and units. Focus is given on numbers in their teens to stamp out the misconception that 13 consist of one and three instead of a ten and three units. They are given sheets of double digit sums to complete on the soroban to continuously think of the partitioning of the 2-digit numbers they are completing. When they are familiar with the partitioning, they are asked to attempt completing two double-digit sums in their heads by imagining how the beads are added on the soroban. Each child will have the soroban with them during this exercise to provide the extra help if needed. Older children within the intervention programme tend to explore place values into hundreds, thousands, and at times millions. They will be asked to name the value of a particular digit within a string of numbers such as the value of the 2 in 2435.

Number sequences: Children work on different number sequences using different starting points to encourage flexibility in counting. I start by showing them how counting works using the soroban. This involves counting forward and backward in ones, tens, hundreds, or thousands. They are, then, asked to complete smaller number sequences such as counting in twos, threes, and fours mentally using different starting point to stop rote learning. This exercise is particularly useful for children who struggle to complete even simple sums such as 2 + 3 mentally as finger counting is discouraged.

The order of addition does not matter : Children are shown that 2 + 3 = 3 + 2. The order of adding any numbers in this case does not change the final sums. This is demonstrated in small quantities before applying the concept to larger sums. As the only difference is the order of the addition, it makes sense that they should only count on the smaller number of the two. Until they understand this, they will count on any numbers in the order given as can be seen from the Year 4 child who counted 30 times instead of 25. While it is not ideal to count on in ones for such a large quantity, this demonstrates the lack of understanding in basic counting skills.

Inverse relationship between addition and subtraction: Children explored the inverse relationship between addition and subtraction through exercises such as finding out the difference in two values, for example, between 11 and 24. They are, then, asked to solve questions written in a different format, 11 + __ = 24, which is solved the same way as finding out the difference in values. In both cases, they are encouraged to place the total sum or larger number, 24, on the soroban and take away the smaller number (11), or the number on the other side of the equal sign, to arrive at the answer. Counting the number of steps involved from 11 to 24 is discouraged as the possibility of miscounting the number of steps involved is fairly high. In order to assure them that the answer obtained is correct, they are asked to check whether 11 + 13 = 24.

As part of the no-finger counting policy, they are shown different strategies in completing single-digit addition and subtraction with regrouping.

<u>Addition with regrouping</u>: Three different strategies for solving single digit additions with regrouping are explored. The first method involves using the understanding of the doubling of single digit numbers. In questions such as 7 + 8, children work out the double of sevens to make 14 and add one more to make 15. The second method requires number bond 10 understanding. They partition the 7 into 5 and 2 and leave the 8 as it is, so that 5 + (2 + 8) = 5 + 10 = 15 or partition the 8 instead into 5 + 3. The last method involves partitioning any numbers larger than 5 into 5 plus some units. They partition 7 + 8 as (5 + 2) + (5 + 3). They, then, re-arrange them into (5 + 5) + (2 + 3), which make 10 + 5 = 15. Children are encouraged to implement the methods they are most comfortable instead of finger counting. When they are comfortable with single digit addition with regrouping, they are encouraged to attempt larger number sums up to hundreds and thousands. The strategies have worked successfully with all the children in the intervention groups as they are able to decide on the preferred method. Weaker children tend to choose Method 3 as it allows them to break the numbers into smaller chunks of units to add up. Others tend to opt for the other two methods.

Figure 6: Different addition methods

Solve 7 + 8	**First step**		**Second step**	
Method 1	7 + 7 + 1=		14 + 1 = 15	
Method 2	Partition 7	Partition 8	Partition 7	Partition 8
	(5 + 2) + 8	7 + (3 + 5)	5 + (2 + 8)	(7 + 3) + 5
Method 3	(5 + 2) + (5 + 3)		(5 + 5) + (2 + 3)	

<u>Subtraction with regrouping</u>: The following strategy is taught for subtraction with regrouping. To complete 13 – 6, for example, they are encouraged to partition the 13 into a ten and three units. The analogy given to the children for this method is 'Imagine you go to a shop with £13 in your pocket, a £10 note and £3 coins. You have found something you like which is going to cost £6. How do you pay for it? Is the £3 coins enough to pay for the £6 item, or do you use the £10?' Children understand that they will need to use the £10 to pay for it and leave the £3 in the pocket as the coins are not enough to pay for the item. When they pay for the item, they will get a £4 change which will be placed in the pocket together with the £3 unused coins. This makes £7 left altogether.

This method made mental computations possible without relying on finger counting. In order to perform this method, however, children would need to be familiar with number bonds 10 and understand the partitioning of the 2-digit numbers.

How can the soroban help children in their mathematical journey?

All of the children who participated in the intervention programmes were selected by their class teachers. They can be largely divided into two categories based on their numerical understanding. The first category of children possess a good basic number skills but are unable to see how sums can be completed more efficiently due to the lack of place value and computations strategies understanding. They are usually quite competent in completing additions or subtractions, but rely heavily on finger counting, or by using other marks which will help them keep track of what they are doing. Alternatively, they will readily apply the column additions methods without a firm understanding of what they mean. This group of children tend to succeed in completing sums involving no regrouping. The lack of understanding will only show when they start attempting sums which will require a better grasp of place value concept where borrowing or carrying over is involved. As an example, children often answer questions such as 34 + 47 = 711.

The second category of the children is those who are lacking in basic number concepts and number sense. For example, they struggle with simple forward or backward counting up to 20, or they are unsure of what 'take away' sign means. They often cannot quite quantify a number. This can usually be seen from giving them blank number lines where each line only contains numbers zero and 20. They usually find it hard to quantify where 5, or 10, or 15 is within the number lines.

I find that the majority of children who have participated in the intervention programmes to fall within the first category. They usually respond to intervention fairly quickly within a term on the basis of two-one hour group sessions per week. Children initially find the soroban to be intriguing and strange. However, once they start learning how to use it, they find it to be helpful in providing the structure in completing sums for its hands-on nature.

The soroban enables a smooth acquisition of place value concept even at an early age. When Year 1 children are shown that 13 is made of a ten and three units on the soroban, they grasp this concept quickly. They can see that the 1 represents one group of 10 objects as it sits on the 'Tens' rod and three is represented by the three 'Ones' beads on the 'Units' rod. This understanding is important as it is consistent with how the Arabic numerals are written. It can help in instilling strong place value concept understanding to children whose native languages do not map the Arabic numerals such as the English. As it allows children to visualise the movement of the beads as they are adding or taking away, this can help in improving their mental computations which can subsequently build their confidence in attempting more challenging sums.

In a 10-hour intervention study conducted on a small sample of Year 1 children, excellent progress in computational skills was particularly noticeable within this intervention group. One of the children, for example, lagged behind by 6 months in her numerical understanding against her peers in a standardised age-equivalent test at the pre-intervention stage, scored 8 months ahead after the intervention. The lowest performing child within this group still outperformed their peers' progress in the study (Freeman, Nov 2014).

Similar effects were seen in a Years 3 and 4 intervention study conducted for a period of 22 hours at the end of 2015. The majority of Year 3 children could not attempt more than 5 out of 25 numerical questions at a pre-intervention stage. None of them could answer questions such as 12 + 12. There also seemed to be a lack of confidence which was indicated by many unattempted questions. Following from the intervention, the majority of the Year 3 children attempted all questions mentally where some of them were able to answer questions such as 468 + 243 and 620 + 620 correctly. There was a good indication of increasing confidence among them which I felt was down to the facts that they could finally make sense of the Arabic structure through the soroban.

A similar performance within the Year 4 intervention group was observed. The child who was earlier described to have counted 30 times when posed with the question 25 + 30, scored as well as the best performer in the control group obtaining 90% in the post-intervention test up from 60% at the pre-intervention stage. She stopped counting on her fingers from the day we started intervention and had instead relied on column addition/subtraction during the post-intervention test. Another significant change was also noted in a boy within the intervention group who could only answer 10 + 7 at the pre-intervention stage and scored 28%. He also expressed that he was not sure of numbers beyond a hundred. He subsequently scored 75% at his post-intervention test and completed all sums mentally.

While the Years 3 and 4 intervention programme has worked well with the majority of the children, there were three children who have special needs statement or other underlying social issues who did not perform as well as expected. These children fall within the second broad category. Although they have shown a slight improvement in their numerical ability and an increasing knowledge of place value, the duration of the intervention is not sufficient to allow a satisfactory progress. This could be due to the speed of the intervention conducted to cater to the majority of children during the intervention. On the conclusion of the Years 3 and 4 intervention study, these children have continued for another year in which they have finally shown an increasing mental computation ability of double digit additions with regrouping after a consistent use of the soroban in completing sums.

Conclusion

The soroban has been an invaluable tool used by many Asian children over the centuries. While the popularity of the soroban has been down to the speed and accuracy of mental calculation acquired by children who have used it for a period of time, it can be an excellent tool in conveying the place value concept alone. When introduced early to primary school children, it can be a powerful tool in helping children to see how the place value work which subsequently enable mental computations. This can be particularly useful for children whose first languages do not map the Arabic number structure such as the English. The soroban can be used to demonstrate how a collection of objects beyond ten can be accounted for in different ways which allows children to move beyond the over reliance of the unitary counting. The majority of children who possesses the basic number concepts responded well to the intervention programmes carried out within three-month periods. Those who do not have the basic number concepts and number sense will require a longer intervention period. While the progress is slow with the latter, there is a definite improvement in their understanding following a longer use of the soroban.

Notes

Materials from the intervention and videos will be made available to teachers who would like to participate in the use of the soroban as part of classroom activities for a period of three months. Please email Nisih Freeman at N.freeman@theabacusclub.co.uk to enquire further.

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